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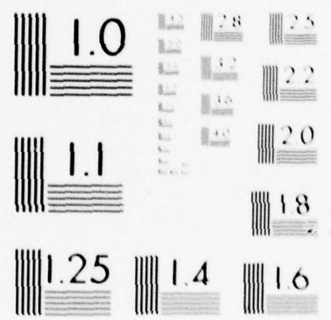

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On Sequential Confidence Intervals for the Largest Normal Mean

Raymond J. Carroll

Institute of Statistics Mimeo Series #1191

August 1978

DEPARTMENT OF STATISTICS
Chapel Hill, North Carolina

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LEVEL II

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On Sequential Confidence Intervals for the Largest Normal Mean

by

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Abstract

We construct a competitor to Tong's method for defining a fixed-width confidence interval for the largest normal mean. This competitor eliminates inferior populations early in the experiment; a Monte-Carlo experiment shows that it can use significantly fewer observations than Tong's method without any real loss in observed coverage probability.

Key Words and Phrases: Monte-Carlo, Largest Mean, Ranking and Selection, Sequential Analysis, Elimination

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1. Introduction

Tong (1975) considered the problem of constructing a fixed-width confidence interval for the largest mean from k normal populations with unknown variance. His procedure is a sequential procedure and is based on the ideas of Chow and Robbins (1965). However, the very nature of this k population problem is qualitatively different from the simpler problem constructing a confidence interval for a mean because the user has the flexibility (not found in Tong's procedure) of sampling selectively from the populations. One method of such selective sampling is *elimination*, where a population is eliminated from further consideration when the data indicate said population is unlikely to be associated with the largest mean.

The purpose of this note is to show that, using the ideas of Swanepoel and Geertsema (1976), it is easy to construct a sequential competitor to Tong's procedure which possesses the elimination option, achieves its intended coverage probability, and can lead to great savings in sample size when the population means are not identical. When the population means are nearly identical, the procedure will take approximately 10% more observations than Tong's procedure, a small price to pay for possible large savings. To this end, in Section 2 we present a Monte-Carlo study of Tong's procedure. In Section 3, we introduce the elimination procedure and study its small sample behavior in a Monte-Carlo study.

2. Tong's Procedure

Suppose we have k populations and take independent and identically distributed observations $X_{i1}, X_{i2}, \ldots$ from population i. Define

$$s_{kn}^2 = (k(n-1))^{-1} \sum_{i=1}^{k} \sum_{j=1}^{n} (X_{ij} - \overline{X}_i^{(n)})^2, \text{ (pooled sample variance)}$$

$$\overline{X}_i^{(n)} = \frac{1}{n} \sum_{j=1}^{n} X_{ij} .$$

We will assume that the observations from the ith population are normally distributed with mean μ_i and common variance σ^2. If $\mu_{[1]} \leq \ldots \leq \mu_{[k]}$ denote the unknown ordered means, the goal is to estimate the largest mean $\mu_{[k]}$ by a confidence interval with coverage probability γ and prescribed fixed length L. Tong's (1973) sequential procedure is of the following form. Let Φ be the standard normal distribution function and define

$$\alpha_k(c,x) = \min\{\Phi^k(c-x) - \Phi^k(-x), \Phi(c-x) - \Phi(-x)\}$$

$$\sup_x \alpha_k(c,x) = \alpha_k(c,x_0(c))$$

$$c_0(\gamma) = \inf\{c: \alpha_k(c,x_0(c)) \geq \gamma\} .$$

He then takes N_T observations from each population, where

$$N_T(L) = \text{smallest integer } n \geq 5 \text{ such that } n \geq (c_0(\gamma)\ s_{kn}/L)^2$$

and announces the following confidence interval of length L:

$$I(N_T(L)) = (\overline{X}_{N_T} - (L - x_0\ s_{N_T}/N_T^{1/2}),\ \overline{X}_{N_T} + x_0\ s_{N_T}/N_T^{1/2}),$$

where $x_0 = x_0(c_0(\gamma))$. Tong shows that

$$\lim_{L\to 0} \frac{N_T(L)}{(c_0(\gamma)\ \sigma/L)^2} \to 1 \quad \text{(almost surely)}$$

$$\lim_{L\to 0} P\{\mu_{[k]} \in I(N_T(L))\} \geq \gamma \quad \text{for all } \mu_1,\ldots,\mu_k \text{ and } \sigma .$$

In Table 1 we present values of $c_0 = c_0(\gamma)$ and $x_0 = x_0(c_0(\gamma))$ for various values of γ and k.

As part of this study we decided to investigate the small sample bahavior of Tong's rule by means of a Monte-Carlo study. We studied the following configurations of means

$$\mu_1 = \mu_2 = \cdots = \mu_{k-1} = \mu_k - d$$

$$\mu_{i+1} - \mu_i = d \quad (i = 1,\ldots,k-1)$$

with d = 0.0, 0.5, 2.0. We chose γ = .90 and L = 0.50, 1.00, with k = 2,3,5 and 10. The results are reported in Tables 2 and 3. It appears that Tong's procedure does indeed approximately achieve it prescribed coverage probability. Note however that the average total number of observations is independent of d, the spacing of the means. It is this undesirable feature of Tong's procedure which we will attempt to improve upon in the next section.

3. Elimination Procedures

The previous section makes clear that while Tong's procedure achieves its coverage probability in small samples, it is "data-blind" in the sense that it takes no account of information available from the data about the relative differences among the means. In order to begin to design a procedure which will take the data more fully into account we investigate a procedure which attempts to eliminate early in the experiment populations which are obviously not associated with the largest mean.

The idea is based upon a technique due to Swanepoel and Geertsema (1976). Essentially, if we desire a coverage probability γ and set $(1-\gamma) = (1-\gamma_0) + (1-\beta)$, we will use Swanepoel and Geertsema's technique (with a minimal sample size of 5), to eliminate populations with error probability at most $(1-\beta)$, and we will use Tong's procedure with coverage probability γ_0 and the remaining populations.

When $\gamma = .90$, we will choose $\gamma_0 = .92$ and $\beta = .98$. This simple grafting of two techniques will result in a procedure which is slightly conservative when the means are all nearly the same but is very efficient when some observations should be eliminated. We outline the steps in the grafting as follows:

Step #1. For any β (typically $.90 \le \beta \le .99$) and k, choose values of (a,t), where

$$t = .2(1 + a^2/4)^5$$

$$1 - F_4(a) + a\, f_4(a) = (1-\beta)/(k-1) ,$$

and $F_4(f_4)$ is the distribution (density) function of a t distribution with four degrees of freedom. The values of (a,t) are given in Table 4.

Step #2. Define

$$H^2(i,j,n) = \frac{1}{n-1} \sum_{p=1}^{n} (X_{ip} - X_{jp} - \overline{X}_i^{(n)} + \overline{X}_j^{(n)})^2$$

$$h(\beta,n) = [(tn)^{1/n} - 1]^{\frac{1}{2}} .$$

We say that population i is *eliminated* at stage $n \ge 5$ if it has not been eliminated before stage n and if

$$\overline{X}_j^{(n)} - \overline{X}_i^{(n)} > h(\beta,n)\, H(i,j,n)$$

for some population j which has yet to be eliminated at stage n.

Step #3. Choose γ as the intended coverage probability. Let $(1-\beta) + (1-\gamma_0) = 1-\gamma$. Take five observations from each population. Use the Tong procedure with γ_0 if $N_T = 5$.

Step #4. If $N_T \ne 5$, eliminate whatever populations you can. Suppose there are k_6 populations left.

Step #5. Take another observation on each remaining population, so that there are now n observations on k_n populations. Compute c_0 and x_0 as in Section 2 of this paper based on γ_0 and k_n. Compute $s^2(n,k_n)$ = pooled sample variance on the k_n populations.

Step #6. If $n \geq (c_0\ s(n,k_n)/L)^2$, discontinue sampling and announce the Tong confidence interval with k_n populations.

Step #7. Otherwise, set $n = n+1$ and see if any more populations can be eliminated. There are now k_n populations left. Return to step #5.

In Tables 5-8 we present the results of a simulation (with 200 iterations) of the elimination rule for a fixed-width confidence interval of length L for the largest normal mean. The prescribed confidence level is $\gamma = .90$, and we chose $\gamma_0 = .92$, $\beta = .98$, so that only obviously inferior populations were eliminated from consideration. In Tables 9 and 10 we present values of the ratio

$$\frac{\text{total observations used by Tong's procedure}}{\text{total observations used by the elimination procedure}} .$$

The tables make clear the following conclusions:

(1) The elimination rule achieves its intended coverage probability.

(2) The elimination rule results in approximately 10% more observations in the case that the means are relatively close and k is small.

(3) When the means differ to any appreciable degree and for larger k, the elimination rule can result in substantial savings in the number of observations taken. For example if $k = 10$, $L = .50$, $d = .50$ and

$$\begin{aligned} \mu_2 - \mu_1 &= d = .50 \\ \mu_3 - \mu_2 &= d = .50 \\ &\vdots \\ \mu_{10} - \mu_9 &= d = .50 , \end{aligned}$$

the elimination procedure takes only 35% of the total observations needed by Tong's procedure, a dramatic savings.

4. Conclusion

We have discovered by a simple grafting technique a procedure which eliminates obviously inferior populations early in the experiment, thus leading to possibly dramatic savings in sample size over the conventional procedure. We argue that elimination methodology is easy to use and easy to study, and that the savings in sample size argue for their implementation.

5. Acknowledgement

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TABLE I

VALUES OF C0 (UPPER ENTRY) AND X0 (LOWER ENTRY) FOR THE LARGEST NORMAL MEAN

γ: / K:	0.900	0.905	0.910	0.915	0.920	0.925	0.930	0.935	0.940	0.945	0.950	0.955	0.960	0.965	0.970	0.975	0.980	0.985	0.990
2	3.290	3.339	3.391	3.445	3.501	3.561	3.624	3.691	3.762	3.838	3.920	4.009	4.107	4.217	4.340	4.483	4.653	4.865	5.152
	1.645	1.670	1.695	1.722	1.751	1.780	1.812	1.845	1.881	1.919	1.960	2.005	2.054	2.108	2.170	2.241	2.326	2.432	2.576
3	3.329	3.379	3.430	3.484	3.541	3.600	3.663	3.730	3.801	3.877	3.959	4.048	4.146	4.255	4.378	4.521	4.690	4.901	5.187
	1.509	1.535	1.562	1.590	1.620	1.651	1.684	1.718	1.755	1.795	1.838	1.884	1.935	1.992	2.056	2.130	2.218	2.327	2.475
4	3.390	3.439	3.491	3.545	3.601	3.660	3.723	3.789	3.860	3.936	4.017	4.106	4.204	4.312	4.435	4.576	4.744	4.954	5.239
	1.447	1.473	1.501	1.530	1.560	1.592	1.626	1.661	1.699	1.740	1.783	1.831	1.883	1.941	2.006	2.082	2.171	2.283	2.433
5	3.447	3.496	3.548	3.601	3.658	3.717	3.779	3.845	3.915	3.991	4.072	4.160	4.257	4.365	4.487	4.627	4.795	5.004	5.286
	1.411	1.438	1.466	1.496	1.526	1.559	1.593	1.629	1.668	1.709	1.753	1.801	1.854	1.913	1.979	2.055	2.145	2.258	2.409
6	3.498	3.547	3.598	3.652	3.708	3.766	3.828	3.894	3.964	4.039	4.120	4.208	4.304	4.411	4.533	4.672	4.839	5.047	5.328
	1.388	1.415	1.444	1.474	1.505	1.537	1.572	1.609	1.647	1.689	1.734	1.782	1.835	1.894	1.961	2.038	2.129	2.242	2.394
7	3.544	3.592	3.643	3.696	3.752	3.810	3.872	3.938	4.007	4.082	4.162	4.250	4.346	4.452	4.573	4.712	4.878	5.085	5.365
	1.372	1.399	1.428	1.458	1.490	1.523	1.557	1.594	1.633	1.675	1.720	1.769	1.823	1.882	1.949	2.026	2.117	2.231	2.384
8	3.584	3.632	3.683	3.736	3.791	3.850	3.911	3.976	4.046	4.120	4.200	4.287	4.383	4.489	4.609	4.748	4.913	5.119	5.398
	1.360	1.388	1.417	1.447	1.479	1.512	1.547	1.584	1.623	1.665	1.710	1.759	1.813	1.873	1.940	2.017	2.109	2.223	2.376
9	3.620	3.668	3.719	3.771	3.827	3.885	3.946	4.011	4.080	4.154	4.234	4.321	4.416	4.522	4.642	4.780	4.944	5.150	5.428
	1.351	1.379	1.408	1.438	1.470	1.503	1.538	1.576	1.615	1.657	1.703	1.752	1.806	1.866	1.933	2.010	2.102	2.217	2.370
10	3.652	3.701	3.751	3.804	3.859	3.916	3.978	4.042	4.111	4.185	4.264	4.351	4.446	4.552	4.671	4.809	4.973	5.177	5.455
	1.344	1.372	1.401	1.431	1.463	1.497	1.532	1.569	1.609	1.651	1.697	1.746	1.800	1.860	1.928	2.005	2.097	2.212	2.366

Table 2

Average number of correct decisions in 200 simulations of Tong's procedure,

$\mu_1 = \ldots = \mu_{k-1} = \mu_{k-d}$

k	2	3	5	10
d = 0.0, L = .50	.900	.900	.930	.905
d = 0.0, L = 1.0	.905	.910	.915	.890
d = .50, L = .50	.890	.930	.930	.920
d = .50, L = 1.0	.930	.925	.975	.980
d = 2.0, L = .50	.890	.925	.920	.900
d = 2.0, L = 1.0	.900	.865	.910	.890

N.B. The average sample size × k (the total number of observations) upon stopping is

k	2	3	5	10
L = 1.00	21	33	59	137
L = .50	84	132	238	538

Table 3

Average number of correct decisions in 200 simulations of Tong's procedure, $\mu_{i+1} - \mu_i = d \quad (i = 1, \ldots, k-1)$.

k	2	3	5	10
d = 0.0 L = .50	.900	.900	.930	.905
d = 0.0 L = 1.0	.905	.910	.915	.890
d = .50 L = .50	.890	.925	.925	.900
d = .50 L = 1.0	.930	.890	.955	.930
d = 2.0 L = .50	.890	.925	.920	.900
d = 2.0 L = 1.0	.900	.865	.910	.890

Table 4

SEQUENTIAL ESTIMATION OF LARGEST MEAN: ELIMINATION PROCEDURE
VALUES OF A (UPPER ENTRY) AND T (LOWER ENTRY) FOR LARGEST K NORMAL MEANS

β \ K:	2	3	4	5	6	7	8	9	10
0.900	0.27502D 01	[illegible] 01	0.40533D 01	[illegible] 01	0.47613D 01	0.50289D 01	0.52620D 01	0.54715D 01	0.56605D 01
	[illegible] 02	[illegible] 03	[illegible] 03	[illegible] 04	[illegible] 04	0.42106D 04	0.62496D 04	0.87930D 04	0.11878D 05
0.905	[illegible] 01	[illegible] 01	0.41274D 01	[illegible] 01	[illegible] 01	0.51600D 01	0.53423D 01	0.55532D 01	0.57443D 01
	[illegible] 02	[illegible] 03	0.80443D 03	0.16954D 04	0.30075D 04	0.48124D 04	0.71260D 04	0.10024D 05	0.13558D 05
0.910	[illegible] 01	[illegible] 01	0.41993D 01	0.45537D 01	0.49145D 01	0.51688D 01	0.54270D 01	0.56403D 01	0.58336D 01
	0.51152D 02	[illegible] 03	0.92557D 03	0.19467D 04	0.34560D 04	0.55156D 04	0.81623D 04	0.11507D 05	0.15558D 05
0.915	0.27102D 01	[illegible] 01	0.42760D 01	0.46747D 01	0.49994D 01	0.52758D 01	0.55176D 01	0.57335D 01	0.59291D 01
	0.59700D 02	[illegible] 03	0.10708D 04	0.22555D 04	0.40021D 04	0.63849D 04	0.94687D 04	0.13313D 05	0.17974D 05
0.920	[illegible] 01	[illegible] 01	[illegible] 01	0.47615D 01	0.50902D 01	0.53699D 01	0.56149D 01	0.58336D 01	0.60317D 01
	[illegible] 02	[illegible] 03	[illegible] 04	[illegible] 04	0.46752D 04	0.74553D 04	0.11054D 05	0.15538D 05	0.20973D 05
0.925	0.30612D 01	[illegible] 01	0.44466D 01	0.48531D 01	0.51860D 01	0.54715D 01	0.57196D 01	0.59415D 01	0.61425D 01
	0.85487D 02	[illegible] 03	0.14854D 04	0.31120D 04	0.55156D 04	0.87930D 04	0.13032D 05	0.18315D 05	0.24716D 05
0.930	0.31419D 01	[illegible] 01	0.45427D 01	0.49584D 01	0.52946D 01	0.55816D 01	0.58336D 01	0.60586D 01	0.62626D 01
	0.10032D 03	[illegible] 03	0.17725D 04	0.37152D 04	0.65312D 04	0.10487D 05	0.15538D 05	0.21831D 05	0.29456D 05
0.935	0.32254D 01	[illegible] 01	0.46471D 01	0.50669D 01	0.54096D 01	0.57017D 01	0.59577D 01	0.61864D 01	0.63937D 01
	0.12215D 03	[illegible] 03	0.21455D 04	0.44930D 04	0.79544D 04	0.12670D 05	0.18766D 05	0.26360D 05	0.35560D 05
0.940	0.33247D 01	0.41559D 01	0.47616D 01	[illegible] 01	0.55365D 01	0.58336D 01	0.60940D 01	0.63267D 01	0.65377D 01
	0.15106D 03	0.92557D 03	0.26526D 04	0.55158D 04	0.97593D 04	0.15558D 05	0.23007D 05	0.32308D 05	0.43575D 05
0.945	[illegible] 01	[illegible] 01	[illegible] 01	[illegible] 01	0.56768D 01	0.59795D 01	0.62446D 01	0.64820D 01	0.66971D 01
	[illegible] 03	0.11595D 04	0.32970D 04	0.68910D 04	0.12186D 05	0.19393D 05	0.28705D 05	0.40300D 05	0.54341D 05
0.950	[illegible] 01	[illegible] 01	[illegible] 01	0.54715D 01	0.58336D 01	0.61425D 01	0.64134D 01	0.66557D 01	0.68754D 01
	0.24416D 03	0.14685D 04	0.42106D 04	0.87930D 04	0.15558D 05	0.24716D 05	0.36572D 05	0.51329D 05	0.69198D 05
0.955	0.36762D 01	[illegible] 01	0.51600D 01	[illegible] 01	0.60106D 01	0.63267D 01	0.66040D 01	0.68519D 01	0.70769D 01
	0.32190D 03	0.19467D 04	0.55156D 04	[illegible] 05	0.20321D 05	0.32308D 05	0.47789D 05	0.67052D 05	0.90375D 05
0.960	[illegible] 01	0.47615D 01	0.53699D 01	[illegible] 01	0.62135D 01	0.65377D 01	0.68223D 01	0.70769D 01	0.73080D 01
	[illegible] 03	[illegible] 04	0.74553D 04	[illegible] 05	0.27426D 05	0.43575D 05	0.64426D 05	0.90375D 05	0.12177D 06
0.965	[illegible] 01	[illegible] 01	0.55616D 01	[illegible] 01	0.64496D 01	0.67837D 01	0.70769D 01	0.73394D 01	0.75776D 01
	[illegible] 03	[illegible] 04	0.10487D 05	[illegible] 05	[illegible] 05	0.61146D 05	0.90373D 05	0.12672D 06	0.17071D 06
0.970	0.41993D 01	[illegible] 01	[illegible] 01	[illegible] 01	0.67312D 01	0.70769D 01	0.73806D 01	0.76524D 01	0.78992D 01
	0.92557D 03	[illegible] 04	[illegible] 05	[illegible] 05	0.56930D 05	0.90373D 05	0.13551D 06	0.18715D 06	0.25205D 06
0.975	[illegible] 01	0.54715D 01	0.61425D 01	0.66557D 01	0.70769D 01	0.74373D 01	0.77538D 01	0.80373D 01	0.82947D 01
	0.14685D 04	[illegible] 04	0.24716D 05	[illegible] 05	0.90373D 05	0.14338D 06	0.21172D 06	0.29668D 06	0.39942D 06
0.980	0.47615D 01	[illegible] 01	[illegible] 01	0.70769D 01	0.75200D 01	0.78992D 01	0.82325D 01	0.85310D 01	0.88021D 01
	[illegible] 04	[illegible] 05	[illegible] 05	[illegible] 05	0.15597D 06	0.25205D 06	0.37201D 06	0.52107D 06	0.70129D 06
0.985	[illegible] 01	[illegible] 01	0.71769D 01	0.76524D 01	0.81257D 01	0.85310D 01	0.88673D 01	0.92067D 01	0.94968D 01
	[illegible] 04	[illegible] 05	0.90373D 05	0.18715D 06	0.32509D 06	0.52107D 06	0.76864D 06	0.10762D 07	0.14479D 07
0.990	0.58336D 01	0.70769D 01	0.78992D 01	0.85310D 01	0.90511D 01	0.94968D 01	0.98889D 01	0.10240D 02	0.10560D 02
	0.15558D 05	0.90375D 05	0.25205D 06	0.52107D 06	0.91463D 06	0.14479D 07	0.21344D 07	0.29869D 07	0.40168D 07

Table 5

Coverage probabilities for elimination rule with $\gamma = .90$ and $\mu_1 = \cdots = \mu_{k-1} = \mu_k - d$.

k	2	3	5	10
d = 0.0 L = .50	.930	.930	.915	.905
d = 0.0 L = 1.0	.920	.890	.915	.920
d = .50 L = .50	.895	.950	.910	.925
d = .50 L = 1.0	.950	.940	.980	.980
d = 2.0 L = .50	.880	.935	.895	.890
d = 2.0 L = 1.0	.880	.880	.915	.910

Table 6

Average total number of observations for elimination rule with γ = .90 and

$\mu_1 = \ldots = \mu_{k-1} = \mu_k - d$.

k	2	3	5	10
k = 0.0 L = .50	94	147	264	593
d = 0.0 L = 1.0	23	36	67	152
d = .50 L = .50	88	141	253	575
d = .50 L = 1.0	23	36	67	152
d = 2.0 L = .50	55	67	90	160
d = 2.0 L = 1.0	19	30	53	120

Table 7

Coverage probabilities for elimination rule with γ = .90 and $\mu_{i+1} - \mu_i = d$, $i = 1,\ldots,k-1$.

k	2	3	5	10
d = 0.0 L = .50	.930	.930	.915	.905
d = 0.0 L = 1.0	.920	.890	.915	.920
d = .50 L = .50	.895	.940	.895	.910
d = .50 L = 1.0	.950	.946	.960	.930
d = 2.0 L = .50	.880	.935	.900	.885
d = 2.0 L = 1.0	.880	.875	.875	.910

Table 8

Average total number of observations for elimination rule with $\gamma = .90$ and $\mu_{i+1} - \mu_i = d$, $i = 1,\ldots,k-1$.

k	2	3	5	10
d = 0.0 L = .50	94	147	264	593
d = 0.0 L = 1.0	23	36	67	152
d = .50 L = .50	88	120	150	189
d = .50 L = 1.0	23	36	61	97
d = 2.0 L = .50	55	62	73	98
d = 2.0 L = 1.0	19	25	36	62

Table 9

Ratio of Tong's total sample size to elimination rule's total sample size for $\mu_1 = \ldots = \mu_{k-1} = \mu_k - d$.

k	2	3	5	10
d = 0.0 L = .50	.89	.90	.90	.91
d = 0.0 L = 1.0	.91	.92	.88	.90
d = .50 L = .50	.95	.94	.94	.94
d = .50 L = 1.0	.91	.92	.88	.90
d = 2.0 L = .50	1.53	1.97	2.64	3.36
d = 2.0 L = 1.0	1.11	1.10	1.11	1.14

Table 10

Ratio of TOng's total sample size to elimination rule's total sample size for $\mu_{i+1} - \mu_i = d,\ i = 1,\dots,k-1.$

k	2	3	5	10
d = 0.0 L = .50	.89	.90	.90	.91
d = 0.0 L = 1.0	.91	.92	.88	.90
d = .50 L = .50	.95	1.1	1.59	2.85
d = .50 L = 1.0	.91	.92	.97	1.41
d = 2.0 L = .50	1.53	2.13	3.26	5.49
d = 2.0 L = 1.0	1.11	1.32	1.64	2.21